SOME DISCRETE DISTRIBUTIONS

Name	**Genesis**	**Notation**	**p.f.**	**E(X)**	**V(X)**	**Applications**	**Comments**
Uniform (discrete)	Set of k equally likely outcomes (usually, not necessarily, the integers)	U$(1, ..., k)$ (not standard)	$p(x) = 1/k$ $x = 1, ..., k$	$\frac{k+1}{2}$	$\frac{k^2-1}{12}$	Dice	
Bernoulli trial	Expt. with two outcomes: 'success' w.p. θ and 'failure' w.p. $1-\theta$ $X \equiv$ no. successes	Ber(θ)	$p(x) = \theta^x(1-\theta)^{1-x}$ $x = 0, 1$ $\theta \in [0, 1]$	θ	$\theta(1-\theta)$	Coins, constituent of more complex distributions	
Binomial	$X \equiv$ no. successes in n ind. $Ber(\theta)$ trials	Bi(n, θ)	$p(x) = \binom{n}{x}\theta^x(1-\theta)^{n-x}$ $x = 0, 1, 2, ..., n$ $\theta \in [0, 1]$	$n\theta$	$n\theta(1-\theta)$	Sampling with replacement	Bi$(1, \theta) \equiv$ Ber(θ)
Geometric	$X \equiv$ no. failures until 1st success in sequence of ind. Ber(θ) trials	Ge(θ)	$p(x) = \theta(1-\theta)^x$ $x = 0, 1, 2, ...$ $\theta \in [0, 1]$	$\frac{1-\theta}{\theta}$	$\frac{1-\theta}{\theta^2}$	Waiting times (for single events)	Alternative formulation in terms of $Y \equiv$ no. of trials to 1st success $(Y = X + 1)$
Negative binomial (or Pascal)	$X \equiv$ no. failures to mth success in sequence of ind. Ber(θ) trials. Generalization of Geometric	Neg Bi(m, θ) (not standard)	$p(x) = \binom{m+x-1}{x}\theta^m(1-\theta)^x$ $x = 0, 1, 2, ...$ $\theta \in [0, 1]$	$\frac{m(1-\theta)}{\theta}$	$\frac{m(1-\theta)}{\theta^2}$	Waiting times (for compound events)	Neg Bi$(1, \theta) \equiv$Ge(θ) Remains valid for any $k > 0$ (not necessarily integer). Alternative formulation as above.
Hypergeometric	$X \equiv$ no. of defectives in sample of size n taken without replacement from population of size N of which d are defective	Hypergeom(N, d, n) (not standard, esp. order of arguments)	$p(x) = \frac{\binom{d}{x}\binom{N-d}{n-x}}{\binom{N}{n}}$ $x = \max(0, n+d-N), ...,$ $..., \min(n, d)$	$\frac{nd}{N}$	$\frac{N-n}{N-1} n \frac{d}{N}\left(1 - \frac{d}{N}\right)$	Sampling without replacement	Sampling with replacement leads to the Bi$(n, \frac{d}{N})$ - a suitable approx if $\frac{n}{N} < 0.1$
Poisson	Arises empirically or via Poisson Process (PP) for counting events. For PP rate ν the no. of events in time $t \sim$Po(νt). Also as an approx. to the Binomial	Po(λ)	$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $x = 0, 1, 2, ...$ $\lambda > 0$	λ	λ	Counting events occurring 'at random' in space or time	Bi$(n, \theta) \equiv$Po$(n\theta)$ if n large, θ small

SOME CONTINUOUS DISTRIBUTIONS

Name	**Notation**	**p.d.f.**	**E(X)**	**V(X)**	**Applications**	**Comments**
Uniform (continuous) (or Rectangular)	$\mathrm{Un}(\alpha,\beta)$	$f(x)=\dfrac{1}{\beta-\alpha}$ $x\in[\alpha,\beta]$ $\alpha<\beta$	$\dfrac{\alpha+\beta}{2}$	$\dfrac{(\beta-\alpha)^2}{12}$	Rounding errors $Un(-\frac{1}{2},\frac{1}{2})$. Simulating other distributions from $Un(0,1)$.	
Exponential	$\mathrm{Ex}(\lambda)$	$f(x)=\lambda e^{-\lambda x}$ $x>0$ $\lambda>0$	$\dfrac{1}{\lambda}$	$\dfrac{1}{\lambda^2}$	Inter-event times for Poisson Process. Models lifetimes of non-ageing items.	Alternative parameterization in terms of $1/\lambda$ $\mathrm{Ga}(1,\lambda)\equiv\mathrm{Ex}(\lambda)$
Gamma	$\mathrm{Ga}(\alpha,\beta)$	$f(x)=\dfrac{\beta^\alpha x^{\alpha-1}e^{-\beta x}}{\Gamma(\alpha)}$ $x\geq 0$ $\alpha,\beta>0$	$\dfrac{\alpha}{\beta}$	$\dfrac{\alpha}{\beta^2}$	Times between k events for Poisson Process. Lifetimes of ageing items.	Alternative parameterization in terms of $1/\beta$ $\mathrm{Ga}(1,\lambda)\equiv\mathrm{Ex}(\lambda)$, $\mathrm{Ga}(\nu/2,1/2)\equiv X^2_\nu$,
Beta	$\mathrm{Be}(\alpha,\beta)$	$f(x)=\dfrac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$ $x\in[0,1]$ $\alpha,\beta>0$	$\dfrac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$	Useful model for variables with finite range. Also as a Bayesian conjugate prior.	$\mathrm{Be}(1,1)\equiv\mathrm{Un}(0,1)$ $\mathrm{Be}(\alpha,\beta)$ is reflection about $\frac{1}{2}$ of $\mathrm{Be}(\beta,\alpha)$. Can transform $\mathrm{Be}(\alpha,\beta)$ on $[0,1]$ to any finite range $[a,b]$ by $Y=(b-a)X+a$
Normal	$\mathrm{N}(\mu,\sigma^2)$	$f(x)=\dfrac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\dfrac{x-\mu}{\sigma}\right)^2\right]$ $x\in(-\infty,\infty)$	μ	σ^2	Empirically and theoretically (via CLT etc.) a good model in many situations. Often easy to handle mathematically.	$X\sim\mathrm{N}(\mu,\sigma^2)\implies$ $aX+b\sim\mathrm{N}(a\mu+b,a^2\sigma^2)$ $\implies Z=\frac{X-\mu}{\sigma}\sim\mathrm{N}(0,1)$ So $P[X\in(u,v)]=P[Z\in\left(\frac{u-\mu}{\sigma},\frac{v-\mu}{\sigma}\right)]$ $\mathrm{N}(0,1)$ special case has p.d.f. denoted ϕ, c.d.f. Φ (tabulated). Note $\Phi(-z)=1-\Phi(z)$.
Chi-square	χ^2_ν	$f(x)=2^{-\nu/2}\Gamma(\alpha)^{-1}x^{\nu/2-1}e^{-x/2}$ $x>0$ $\nu>0$	ν	2ν	Sum of squares of ν standard normals	$X^2_\nu\equiv\mathrm{Ga}(\nu/2,1/2)$ If $X_1,X_2,\ldots,X_n\sim\mathrm{N}(0,1)$ independent, then $\sum_{i=1}^n X_i^2\sim\chi^2_n$
Student t	t_ν	$f(x)=$ $\nu^{-1/2}B\left(\frac{1}{2},\frac{\nu}{2}\right)^{-1}\left(1+x^2/\nu\right)^{-(\nu+1)/2}$ $x\in(-\infty,\infty)$ $\nu>0$	0 (if $\nu>1$)	$\dfrac{\nu}{\nu-2}$ (if $\nu>2$)	Useful alternative to Normal for variables with heavy tails.	If $X\sim\mathrm{N}(0,1)$ and $Y\sim\chi^2_\nu$ independent then $\frac{X}{\sqrt{Y/\nu}}\sim\mathrm{t}_\nu$. $\mathrm{t}_1\equiv$ Cauchy. $\mathrm{t}^2_\nu\equiv\mathrm{F}_{1,\nu}$.

F	$\mathrm{F}_{\nu,\delta}$	$f(x)=\dfrac{\nu^{\nu/2}\delta^{\delta/2}x^{\nu/2-1}}{B(\nu/2,\delta/2)(\nu x+\delta)^{(\nu+\delta)/2}}$
		 $x>0$
		 $\nu,\delta>0$
			$\dfrac{\delta}{\delta-2}$
			 (if $\delta>2$)
				$\frac{2\delta^2(\nu+\delta-2)}{\nu(\delta-2)^2(\delta-4)}$
				 (if $\delta>4$)
					Scaled ratio of chi-squares. Used in tests to compare variances	If $X\sim\chi^2_\nu$ and $Y\sim\chi^2_\delta$ independent then
						 $\frac{X/\nu}{Y/\delta}\sim\mathrm{F}_{\nu,\delta}$.
						 If $T\sim\mathrm{t}_\nu$ then $T^2\sim\mathrm{F}_{1,\nu}$.
						 If $Z\sim\mathrm{Be}(\alpha,\beta)$ then
						 $\frac{\beta Z}{\alpha(1-Z)}\sim\mathrm{F}_{2\alpha,2\beta}$.