Some relations between standard distributions

Notation from distributions handout. Some relations are recorded there too.

Relations to the normal

1. If Z is $\mathrm{N}(0,1)$, then Z^2 is χ^2_1.
2. If $Z \sim \mathrm{N}(0,1)$ is independent of $W \sim \chi^2_\nu$, then

   $$T = \frac{Z}{\sqrt{W/\nu}} \sim \mathrm{t}_\nu,$$

   i.e. has a t-distribution with ν degrees of freedom.
3. If $Z_1, Z_2, ..., Z_n$ are independent $\mathrm{N}(0,1)$ r.v's, then

   $$Z_1^2 + Z_2^2 + ... + Z_n^2 \sim \chi^2_n.$$

4. If $X_1, X_2, ..., X_n$ are independent $\mathrm{N}(\mu, \sigma^2)$ and $\overline{X} = \frac{1}{n}\sum_1^n X_i$ then

   $$\overline{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right).$$

5. If $X_1, X_2, ..., X_n$ are independent $\mathrm{N}(\mu, \sigma^2)$ then

   $$\frac{(X_1 - \overline{X})^2 + (X_2 - \overline{X})^2 + ... + (X_n - \overline{X})^2}{\sigma^2} \sim \chi^2_{n-1}$$

   and it is independent of $\overline{X}$.
6. If $X_1, X_2, ..., X_n$ are independent $\mathrm{N}(\mu, \sigma^2)$,

   $$\overline{X} = \frac{1}{n}\sum_1^n X_i \quad \text{and} \quad S^2 = \frac{1}{n-1}\sum_1^n (X_i - \overline{X})^2$$

   then, combining 2, 4 and 5,

   $$T = \frac{\overline{X} - \mu}{\sqrt{S^2/n}} \sim \mathrm{t}_{n-1}.$$

7. If $W_1 \sim \chi^2_{\nu_1}$ and $W_2 \sim \chi^2_{\nu_2}$ with W_1, W_2 independent, then

   $$F = \frac{W_1/\nu_1}{W_2/\nu_2} \sim \mathrm{F}_{\nu_1,\nu_2},$$

   i.e. has F-distribution with ν_1, ν_2 degrees of freedom.
8. From 1, 2 and 7 above, if $T \sim \mathrm{t}_\nu$, then $T^2 \sim \mathrm{F}_{1,\nu}$.

Other relations

1. If $X \sim \mathrm{Ga}(a, b)$ then $\lambda X \sim \mathrm{Ga}(a, b/\lambda)$. In particular (with $a = 1$) if $X \sim \mathrm{Ex}(b)$ then $\lambda X \sim \mathrm{Ex}(b/\lambda)$.

2. If $X_i \sim \mathrm{Ga}(a_i, b)$ and are independent, then

$$\sum X_i \sim \mathrm{Ga}\left(\sum a_i, b\right).$$

   In particular ($a_i = \nu(i)/2$, $b = 1/2$), $X_1 \sim \chi^2_{\nu(1)}$, if $X_2 \sim \chi^2_{\nu(2)}, \ldots, X_n \sim \chi^2_{\nu(n)}$ and are independent, then

$$X_1 + X_2 + \ldots + X_n \sim \chi^2_{\nu(1)+\nu(2)+\ldots+\nu(n)}.$$

3. If $Y_i \sim \mathrm{Po}(\mu_i)$, independent then

$$\sum Y_i \sim \mathrm{Po}\left(\sum \mu_i\right).$$

4. If $Y \sim \mathrm{Po}\,(\mu)$ and μ is large than $Y \sim \mathrm{N}(\mu, \mu)$ approximately.